Optimal, Truthful, and Private Securities Lending

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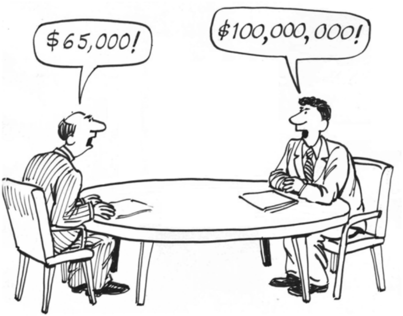
Overview

Motivation

Motivated by challenges associated with securities lending, the mechanism underlying short selling of stocks in financial markets



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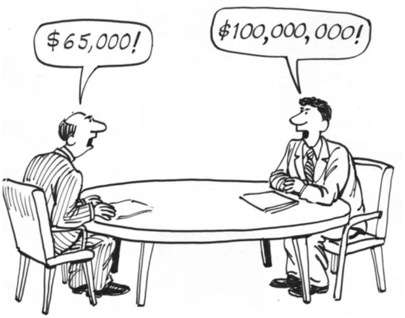

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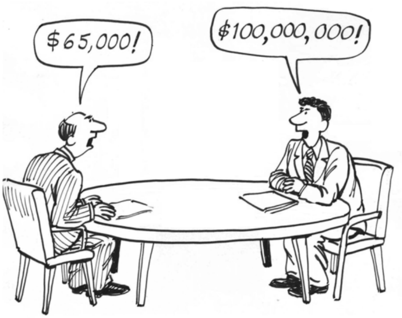

- Consider allocation of a scarce commodity in settings in which privacy concerns or demand uncertainty may be in conflict with truthful reporting
- Want to construct a privacy protecting allocation mechanism that motivates truthful reporting without sacrificing too much utility

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- Client's payoff is number of shares actually used, and lender's utility for allocation rule A is:

$$v(A) = \sum_i \mathbb{E}_{Q_{it},A}[\min(A(r_1, \ldots, r_n; Q_1, \ldots, Q_n)_i, u_{it})]$$

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Table: Sample Truthful Distribution

u_{it} \ r_{it}	0	1	2
0	$\frac{1}{3}$	0	0
1	0	$\frac{1}{3}$	0
2	0	0	$\frac{1}{3}$

Table: Sample Untruthful Distribution

u_{it} \ r_{it}	0	1	2
0	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
1	0	$\frac{1}{6}$	$\frac{1}{6}$
2	0	0	$\frac{1}{3}$

Optimal Allocation Rule

Given knowledge of Q_i, the lender can compute the posterior distribution $Q_i(u_i|r_i)$ on the true demand u_i given r_i, via Bayes' rule:

$$Q_i(u_i|r_i) = \frac{Q_i(r_i|u_i)U_i(u_i)}{\sum_{u'} Q(r_i|u')U_i(u')}$$

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Algorithm 2 Greedy Allocation Rule

Input: $n, \{Q_i(u_i|r_i)\}_{i\in[n]}, V$
Output: feasible allocation $S = \{s_i\}$.
procedure GREEDY($n, \{Q_i(u_i|r_i)\}_{i\in[n]}, V$)
  Initialize $s_i = 0, \ \forall i$. ▷ number of shares allocated to client i
  for $t = 1 \ldots V$ **do**
    Let $i^* = \text{argmax}_i T_i(s_i + 1|r_i)$
    update $s_i \leftarrow s_i + 1$

Optimal Allocation Rule

Theorem: The allocation returned by *Greedy*, S, maximizes the expected payoff for the lender:

$$S \in \arg \max_{S:\sum_i s_i = V} v(S) = \sum_i \mathbb{E}_{Q_i(u|r_i)}[\min(s_i, u_i)]$$

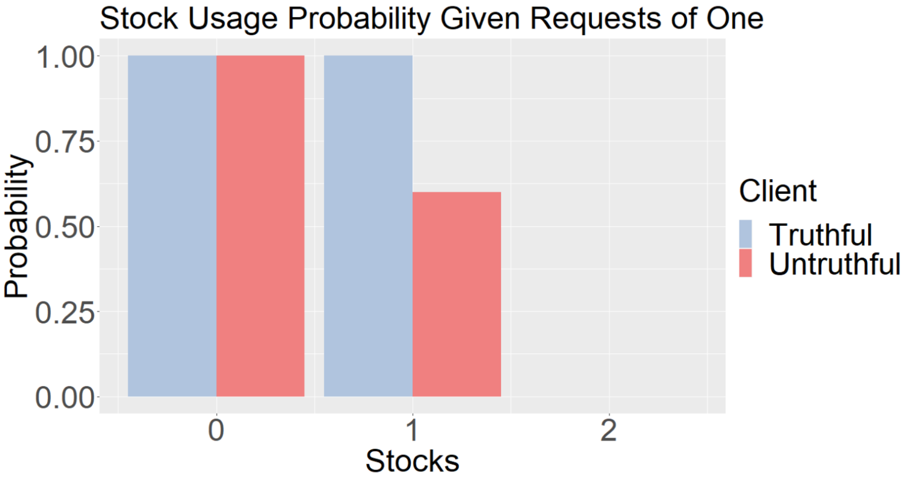
Dominant-Strategy Truthfulness

Given that the lender is solving the allocation problem optimally for the reported Q distributions, truth telling is a dominant strategy:

Theorem: Fix a set of choices Q_{-i} and reports r_{-i} for all clients other than i, and a realization of client i's usage $u_i \sim U_i$. Let Q_i^T denote the truthful strategy $Q_i^T(r_i|u_i) = \mathbf{1}_{r_i}$, and let $Q_i(r_i|u_i)$ denote any other strategy. Let A denote the lender's optimal allocation. Then:

$$v_A^i(Q_i) \leq v_A^i(Q_i^T)$$

Dominant-Strategy Truthfulness



Auction Formulation

- Optimal allocation policy can be implemented as a virtual ascending auction among clients

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- Bidders (clients) have decreasing marginal valuation functions for up to U units of each good (stock)

Algorithm 3 Auction Rule

Input: $\alpha > 0, n, \{v_i\}_{i \in [n]}, U, V$ ▷ valuations $v_i : [U] \to [0, 1]$ satisfy DMR property
Output: feasible allocation S.

procedure AUCTION(α, U, V)
  Initialize array S of length n, $S[i] \leftarrow 0 \forall i$ ▷ goods currently allocated to player i
  Initialize $cB \leftarrow n$, $T_B \leftarrow 0$ ▷ bids in current round, total bids
  Set the price $p = 0$, $m = 1$ ▷ m is index of good currently being allocated
  while $cB \neq 0$ **do** ▷ terminate if there are 0 bids in the round
    $cB \leftarrow 0$
    for $i = 1 \ldots n$ **do**
      Let $\Delta_i = v_i(S[i] + 1) - v_i(S[i])$ ▷ marginal utility of additional good
      if $\Delta_i \geq p$ **then**
        $cB \leftarrow cB + 1, S[i] \leftarrow S[i] + 1, m \leftarrow (m + 1) \pmod{V}$
        $S[i_m] \leftarrow S[i_m] - 1$ ▷ i_m is player holding good m
        if $T_B \pmod{V} = 0$ **then** ▷ increment price every V bids
          $p \leftarrow p + \alpha$
  return S

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  1. Reporting number of bids placed so far with a differentially private estimator
  2. Allowing the algorithm to stop early
  3. Running the auction with $V - E$ shares, where E corresponds to error of differentially private bid counter
- Then, truthful reporting is still an approximately dominant strategy

Approximate Optimality and Truthfulness

Finally, if clients are allowed to adapt strategies with time, joint differential privacy enforces truthfulness as an approximately dominant strategy and guarantees near optimality

Theorem: Let A be a private auction with appropriate values of U, V, ϵ and ρ such that A is $(\epsilon', \beta/T)$-JDP with $\epsilon' = \tilde{O}(\epsilon/\sqrt{T})$ and outputs S such that $E[V(S)] \geq (1-\rho)OPT_V - \rho$. Take β, ρ such that $\sqrt{\beta + (1-\beta)\rho} \leq \beta^2/T$. Then for a $(1-\beta)$ fraction of the n clients i, let L^t_{i*} denote the truthful strategies, and let L^t_i be any other set of strategies. Then a private greedy allocation rule for the private auction satisfies:

$$v_i(L^1_i, \ldots, L^n_i) \leq e^{2\epsilon} v_i(L^1_{i*}, \ldots, L^n_{i*}) + 2\beta UT + e^{\epsilon} \frac{\beta^2}{1-\beta^2/T}$$

$$v_A(L^t_{i*}) \geq (1-\rho)OPT_V - \rho T,$$

where OPT_V denotes the lender's optimal utility.

Summary

- Without privacy constraints, we construct an optimal greedy allocation for which truthfulness is a dominant strategy
- With privacy constraints (joint differential privacy) our allocation mechanism is still nearly optimal and truthfulness is still an approximately dominant strategy

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